Physics Letters A 171 (1992) 17–20
North-Holland

PHYSICS LETTERS A

Appearance of a local world

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Received 20 July 1992; accepted for publication 22 September 1992
Communicated by J.P. Vigier

Why is it that nonlocality effects are not normally observed for pairs of systems one of which is (or both are) macroscopic? This question is investigated in detail for the paradigm case of a measurement process, the instrument being viewed as a quantum system. It is shown that whenever only *classical* dynamical quantities can be measured on the instrument the correlations between the instrument and the measured system cannot exhibit any violation of the local causality assumption. Implications are drawn concerning the *empirical reality* notion.

It is known from Bell's theorem that quantum mechanics entails nonseparability (the violation of local causality) in a variety of cases. Why is it then that nonseparability is never observed except just in experiments that have been conceived for the quite specific purpose of testing it?

Of course, according to its very definition nonseparability can only be seen in phenomena involving correlations between two or more distant physical systems that interacted in the past. This gives a first hint as to where to look for an answer: most experiments in physics are *not* concerned with phenomena of this type. But still, in ordinary life as well as in macroscopic physics distant correlation effects trivially occur in many instances, so that it is a legitimate question to ask why nonseparability effects are not observed there. Would *some* be observed, we may inquire, if actually looked for? Are there theoretical reasons to believe none would? If so, what exactly are these reasons?

There are two quite different general lines along which these questions can be approached. One is to try and show that for such systems local causality cannot even be defined meaningfully, so that the violation of the Bell inequalities and similar tests are void of physical significance. This is impossible within the realm of any theory that aims at being ontologically interpretable (Bell's theorem) but can be achieved if this condition is appropriately relaxed. More precisely, it must then be relaxed in such a way that the very notions of "objective state" and of "elements of reality" (as defined by EPR) are made meaningless at least in so far as the *microscopic* dynamical attributes of the considered systems are concerned [1]. A precise way to do this has been described in the literature [2,3]. Its peculiarity is that it restricts quite drastically the range of the notion of truth as applied to propositions of physics in general, and particularly subatomic physics.

Another approach, the one proposed here, centers less on logical-epistemological and more on physical considerations. It focuses on the measurements that can be done *in practice* on the systems under study, and aims at showing that the outcomes of these measurements *must obey* local causality. In favor of this conjecture there seems to exist some sort of general feeling, sometimes backed up by sketchy arguments. But the matter is important and worth being analyzed step by step.

Although the following considerations would *a fortiori* apply to pairs of correlated macroscopic systems, they are developed here concerning pairs one component, S, of which is a microsystem while the other one, A, is a macroscopic one; and, somewhat more specifically within the assumption that A is an apparatus used for performing the measurement of an observable pertaining to S. As for the definition of what we mean by a "macroscopic" system we take up the one according to which a macroscopic system is a system only the *classical* attributes of which can be measured in practice.

For this definition to be precise the expression "classical attribute" must in turn be defined. Following Omnès [4] let us do this by referring to the classical dynamical variables $a(q_i, p_i)$ $(i=1, ..., n)$ associated with the collective observables of a system by means of Wigner's 1932 formula [5]. Within the $2n$-dimensional phase space $\{q_i, p_i\}$ we may then consider a set of many nonoverlapping cells, C_ν. It can then be shown [4] that

(a) provided their shape is simple enough and their volume much larger than h^n such cells can be associated with projectors (more precisely quasi-projectors) in an appropriate subspace of the overall Hilbert space of the system, and

(b) the commutator of two distinct such projectors is vanishingly small as soon as the corresponding cells are clearly separated. By definition the classical attributes of A are then the observables represented by such commuting projectors (together of course with the observables the spectral decomposition of which involves only these projectors).

Keeping these notions in mind let us recall [6] how they apply to the well-known measurement (or "Schrödinger's cat") riddle. Let G, with eigenvalue equation

$$G|n, r\rangle = g_n|n, r\rangle\,, \tag{1}$$

be the instrument coordinate ("pointer position") of A and let B, with eigenvalue equation

$$B|\varphi_n\rangle = b_n|\varphi_n\rangle\,, \tag{2}$$

be the quantity measured on S. The interaction between S and A is assumed to be such that

$$|\varphi_n\rangle|0, s\rangle \rightarrow |\varphi_n\rangle|n, r\rangle\,, \tag{3}$$

where $|0, s\rangle$ is any one of the instrument states corresponding to $G=g_0$. The above mentioned "measurement riddle" is then that if initially S is in a superposition $\Sigma\, a_n|\varphi_n\rangle$ of states $|\varphi_n\rangle$ the linearity of the Schrödinger time dependent equation entails that the final S+A state is

$$|\psi_f\rangle = \sum_n a_n|\varphi_n\rangle|n, r\rangle\,. \tag{4}$$

According to our general assumptions, since A is a macroscopic system, the only dynamical quantities that can in practice be measured in A are, along with G, quantities described by operators $Q_1, ..., Q_l, ...$, that all commute with G and with one another. Let H^S and H^A be the Hilbert spaces of S and A respectively and in the eigensubspace $\mathscr{E}_n$ of H^A that corresponds to $G=g_n$ let us take as a basis the eigenvectors $|t_1, ..., t_l, ..., n, \nu\rangle$ common to G and to $Q_1, ..., Q_l, ...$:

$$G|t_1, ..., t_l, ..., n, \nu\rangle = g_n|t_1, ..., t_l, ..., n, \nu\rangle\,, \tag{5}$$

$$Q_l|t_1, ..., t_l, ..., n, \nu\rangle = q^{(l)}_{t_l}|t_1, ..., t_l, ..., n, \nu\rangle\,. \tag{6}$$

With a trivial change of basis within each $\mathscr{E}_n$, eq. (4) can be rewritten (with t standing for $\{t_1, ..., t_l, ...\}$)

$$|\psi_f\rangle = \sum_{t,n,\nu} b_{t,n,\nu}|\varphi_n\rangle|t_1, ..., t_l, ..., n, \nu\rangle\,. \tag{7}$$

Let then F be one or other of the observables of S, with eigenvalue equation

$$F|f\rangle = f|f\rangle\,. \tag{8}$$

The probability $p(f, q^{(l)}_{t_l})$ that simultaneous measurements of F on S and Q_l on A yield outcomes f and $q^{(l)}_{t_l}$ is then

$$p(f, q^{(l)}_{t_l}) = \sum_{t,n,\nu}{}' |\langle f|\langle t_1, ..., t_l, ..., n, \nu|\psi_f\rangle|^2\,, \tag{9}$$

$$= \sum_{t,n,\nu}{}' |b_{t,n,\nu}\langle f|\varphi_n\rangle|^2\,, \tag{10}$$

$$= \sum_n |\langle f|\varphi_n\rangle|^2 \sum_{t,\nu}{}' |b_{t,n,\nu}|^2$$

$$= \sum_n h_{n,t_l}|\langle f|\varphi_n\rangle|^2\,, \tag{11}$$

with

$$h_{n,t_l} = \sum_{t,\nu}{}' |b_{t,n,\nu}|^2\,, \tag{12}$$

where in eqs. (9)–(12) Σ' means that the summation does not extend over index t_l. Similarly the probability that simultaneous measurements of G and Q_l yield outcomes g_n and $q^{(l)}_{t_l}$ is just h_{n,t_l} and the probability p_n that a measurement of G yields g_n irrespective of what measurements of the Q_l's yield is

$$p_n = \sum_{t_l} h_{n,t_l}\,. \tag{13}$$

Conversely, the conditional probability $p^{(n)}_{t_l}$ that a measurement of Q_l yields $q^{(l)}_{t_l}$ if it is known that a measurement of G yields (or has yielded) g_n is of course

$$p_{ti}^{(n)} = h_{n,\,ti}\, p_n^{-1}\,. \tag{14}$$

With these notations eq. (11) reads

$$p(f, q_{ti}^{(i)}) = \sum_n p_n p_{ti}^{(n)}\, |\langle f|\varphi_n\rangle|^2\,. \tag{15}$$

In order to extract from this equation some information connected with local causality, let us define, for any value of the index n, an ensemble E_n described by the following (normalized) direct product of a ket belonging to H^S with one belonging to H^A,

$$|\psi_n\rangle = p_n^{-1}\, |\varphi_n\rangle \sum_{t,v} b_{t,n,v}\, |t_1, ..., t_i, ..., n, v\rangle\,.$$

Within E_n the outcomes of a measurement of F on S and a measurement of Q_i on A are manifestly uncorrelated, as is always the case whenever the representative ket is a direct product. The probability that a measurement of F yields outcome f is $|\langle f|\varphi_n\rangle|^2$ and the probability that a measurement of Q_i yields outcome $q_{ti}^{(i)}$ is $p_{ti}^{(n)}$, independently in both cases of whether or not a measurement of the other observable is performed and, if it is, of its outcome. Local causality is therefore obeyed in any E_n. It follows that it is also obeyed in any proper mixture of the E_n's, any correlation between the outcomes being then due to "local causes" differing from one E_n to the other (here these local causes are of course the different values g_n of G). But on the other hand, with the above noted values of the relevant probabilities, inspection of eq. (15) shows that the predictions concerning the outcomes of simultaneous measurements of F on S and Q_i on A are the same on an ensemble E described by $|\psi_f\rangle$ as on a proper mixture with weights p_n of the E_n. It follows that according to our assumption that A is a macrosystem (and in view of the definition of macrosystems that we are using), there can be no observable difference between this proper mixture and ensemble E. Two parallel consequences of this must be stressed. One of them, which was pointed out long ago [6,7], is that as long as the assumption in question can consistently be kept, no experiment can show we are mistaken when we say that in a proportion p_n of the components of E observable G *has* value g_n. The other one, which this article purports to point out, is that under the same general conditions, although local causality *is* violated in E as long as the ket (here $|\psi_f\rangle$) is viewed as a complete description of ensemble E, still this violation can have no observable effect whatsoever.

In other words, all the consequences of local causality must hold true in the present case as long as only quantities "measurable in practice" are considered. This is true in particular concerning the most remarkable of these consequences, namely the Bell inequalities. In fact a simple direct calculation leads to the same result. The mean value of a correlation product of the type FQ_i is

$$\langle FQ_i\rangle = \sum_{t,n,v} |b_{t,n,v}|^2 \langle\varphi_n|F|\varphi_n\rangle q_{ti}^{(i)}\,,$$

which can also be written as

$$\langle FQ_i\rangle = \sum_n p_n \langle F_n\rangle \langle Q_{i,n}\rangle\,, \tag{16}$$

where

$$\langle F_n\rangle = \langle\varphi_n|F|\varphi_n\rangle \tag{17}$$

and

$$\langle Q_{i,n}\rangle = \sum_i p_{ti}^{(n)} q_{ti}^{(i)} \tag{18}$$

are the mean values in E_n of F and Q_i respectively. Expression (16) is identical to the one that, in objective local theories, describes the mean value of a correlation product, n (or g_n) playing the role of the "objective state" λ and p_n that of the "density of objective states" $\rho(\lambda)$. The Bell inequalities follow (of course this does not come as a surprise: we all realize that a violation of these inequalities can be expected only when pairs of incompatible measurements are considered on *both* systems).

Recently Peres [8] and Khalfin and Tsirelson [9] probed theorems that go along the same lines as what has been shown here. Peres could show that although, in a case in which a pair of spin j particles is created in a singlet state, the Bell inequalities remain violated even for arbitrarily large j values, still they are *obeyed* by the outcomes of any measurements in which neighboring values of a J component are lumped together because of limited instrumental resolution. As for Khalfin and Tsirelson, they showed a result rather similar to ours, but with, for the macroscopic systems, a different definition that has the consequence that the Bell inequalities could conceivably be violated, in border-line cases, even for some macroscopic bodies.

To correctly understand the result described here it is necessary to remember that it relies on an avowedly anthropocentric definition of the macroscopic systems and that therefore one and the same system may well be, or not be, macroscopic, according to the degree of refinement of the experiments it is subjected to. For this very reason, however, the result in question is useful as a means of stressing the relevance and further clarifying the nature of a distinction between the concepts of *independent* and *empirical* reality that quantum mechanics seems to suggest [10]. While no description of *independent* reality can be made to accommodate local causality, the fact that nonseparability does not allow for faster-than-light signalling could already be seen as a significant indication that it should be possible to define *empirical* reality in such a way that *it* at least should not violate local causality. The here obtained result shows that indeed this can be done rather simply, just by defining empirical reality as essentially being the set of the macroscopic systems.

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