CONSISTENT HISTORIES AND THE MEASUREMENT PROBLEM

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The physical grounds and significance of Griffiths' "consistent histories" theory are investigated. A simple derivation of t consistency conditions is given. The question is studied whether or not this theory makes it possible to generally consider quantum measurement as informing us of the value the measured quantity *had*, before the measurement was done. It is show that the answer is negative, unless the logical rule known as *modus ponens* is given up, which does not seem advisable.

1. Introduction

The mainstream of quantum interpretation, called "the orthodox view" by Wigner [1], is centered on the notion of measurement. It sees non-relativistic quantum theory essentially as a tool for predicting the probability of the result of one or several successive measurements carried out on a system S, and it expresses this probability by means of the general formula

$$w(a_1, ..., a_{p-1}, a_p)$$

$$= \mathrm{Tr}[E_p^{a_p} E_{p-1}^{a_{p-1}}, ..., E_1^{a_1} \rho E_1^{a_1}, ..., E_{p-1}^{a_{p-1}}] , \qquad (1)$$

where ρ is the statistical operator describing the state of the system S before measurement, $E_k^{a_k}$ is the Heisenberg projection operator onto the eigensubspace corresponding to eigenvalue a_k of the observable A_k (an index differentiating the eigenvalues of a given A_k is not expressed) and $w(a_1, ..., a_p)$ is the probability that if the observables $A_1, A_2, ..., A_p$ are measured on S in this order by means of appropriate instruments the results read on the latter be $a_1, a_2, ..., a_p$. The objectivity of this rule consists in the fact that it is valid for any observer. It is therefore appropriate to call it "intersubjectivity" (or "weak objectivity").

Many physicists, including most experimentalists, do not consider weak objectivity (or equivalently an interpretation of quantum physics centered exclusively on formula (1)) as entirely satisfactory. The would like to be able to consider the result of a me surement as informing us, just as in classical physic of what the value of the measured quantity wa immediately before the measurement in questio Recently, Griffiths proposed a "consistent histories theory [2] that may constitute a step forward in thi direction. It is on this attempt that the present articl bears.

2. "Consistency conditions" and measurement

Griffiths' theory is centered on "consistency con ditions" that this author derives from requiring th validity of eq. (1) as regards the probabilities o physical quantities *having* specified values. Here w derive these conditions from more physical consid erations that also facilitate check of their validity i specific cases.

For this purpose let us consider within the real of the "orthodox view" an experiment X in whic two observables A_1 and F are measured successivel at times t_1 and t_f respectively and in which only th second result is noted, and let us compare it to th experiment Y in which only the second measure ment is made. It is clear that in general the proba bility for obtaining result a_f upon measurement of at t_f is different in the two experiments, for in th first one the state of the system is changed throug

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its interaction with the A_1-measuring instrument. There may however be cases in which these two probabilities are equal. Only in such cases can we say that if, in experiment Y, A_1 *had* been measured at t_1 – with, of course, a definite result obtained – nothing would have been changed as regards the F results. Only in such cases can we therefore be sure that by conventionally saying "at t_1 the as yet unobserved quantity A_1 already *had* a definite value" we are not introducing any inconsistency in the overall description. In other words the condition

$$\sum_{a_1} \mathrm{Tr}[E_f^{a_f} E_1^{a_1} \rho E_1^{a_1}] = \mathrm{Tr}[E_f^{a_f} \rho] \tag{2}$$

is a necessary one for such a statement to have a meaning.

Here Σ_{a_i} mean a sum over all the eigenvalues a_i of A_1. Hence

$$\sum_{a_1} E_1^{a_1} = 1$$

so that

$$\mathrm{Tr}[E_f^{a_f} \rho] = \mathrm{Tr}\left[E_f^{a_f}\left(\sum_{a_1} E_1^{a_1}\right)\rho\left(\sum_{a_1} E_1^{a_1}\right)\right]$$

$$= \sum_{a_1} \mathrm{Tr}[E_f^{a_f} E_1^{a_1} \rho E_1^{a_1}] + \sum_{a_1' \neq a_1} \mathrm{Tr}[E_f^{a_f} E_1^{a_1} \rho E_1^{a_1'}] + \mathrm{c.c.} \tag{3}$$

Consequently condition (2) can also be written

$$\mathrm{Re} \sum_{a_1' \neq a_1} \mathrm{Tr}[E_f^{a_f} E_1^{a_1} \rho E_1^{a_1'}] = 0\,. \tag{4}$$

The argument that leads to condition (2) can easily be generalized to more than two physical quantities and to the case in which the system is filtered at intermediate times. The result is Griffiths' consistency conditions (2.14). Eq. (4) can then be generalized so as to set these conditions in the form (2.19) of Griffiths' paper. The above derivation of the consistency conditions makes their physical meaning clear and gives a simple physical criterion for their validity. It shows that they cannot be met when the insertion of non-read intermediate instruments would change the end results.

2.1. *A difficulty*

The above considerations make it quite clear that in general the consistency conditions can be met in at least two ways, namely when the intermediate physical quantity coincides either with the (time transformed) physical quantity measured for preparing the system or with the (time transformed) physical quantity measured at t_f. Another way of expressing this fact is to say that in general there exist at least two distinct "consistent histories", that is two distinct sequences of definite values of physical quantities at different times. Suppose for example that at time t_0 the S_z component of a free spin-1/2 particle is measured and found equal to 1/2 and that at time t_2 the S_x component is measured and also found equal to 1/2. The foregoing argumentation makes it quite clear that we get a consistent history by saying that at time t_1 ($t_0 < t_1 < t_2$) S_z has value $+1/2$ and another one by saying that at that same time S_x has value $+1/2$. A similar remark holds for the case in which we measure at t_2 the position of a particle that has been accelerated so as to have a definite momentum at time t_0, and in infinitely many other cases. In view of this remark the further development of the theory raises difficult problems. Three mutually exclusive ways may be considered.

2.2. *First possibility*

Following Griffiths' let us call D the event "$S_z = 1/2$ at time t_0", F the event "$S_x = 1/2$ at time t_2", A (Γ) the event "S_z (S_x) has value 1/2 at time t_1". Further let the symbols $\wedge$ and $\supset$ mean "and" and "if... then..." respectively, as in conventional logic. The first possibility is to assert that both propositions

$$(D \wedge F) \supset A \tag{5}$$

and

$$(D \wedge F) \supset \Gamma \tag{6}$$

are true but that

$$(D \wedge F) \supset (A \wedge \Gamma) \tag{7}$$

is not. This essentially is Griffiths' proposal (see his comments on his eqs. (4.9), (4.10) and (4.11)). However, it has been pointed out [3] that no types of logic, not even unconventional ones, allow for proposition $a \wedge b$ to be false when both propositions a and b are true. Hence, in order that the falsity of eq. (7) should not contradict the truth of eqs. (5) and (6) it is necessary to give up what the logicians call "modus ponens" or "the primitive rule of inference", and which is: "*given a and $a \supset b$, to infer b*".

Now is such a renunciation advisable? Most scientists would (wisely) answer that it is not, for it would deprive us from what is perhaps the most essential tool in the exercise of reason. This seems to be a high price to pay for just the intellectual comfort of being able to consider that the quantum observables have in advance the values we observe when we measure them (remember this was the incentive of our quest).

2.3. *Second possibility*

It consists in observing that any attempt at recovering a kind of "realistic" description of what physical systems are before measurements are done on them nowadays implies a renunciation of the idea of strict time-reversal symmetry, the reason being that, in the type of undeterministic physics we now have, the idea in question conflicts with a principle which is deeply ingrained in all of our attempts at describing things realistically, namely the principle that the future cannot influence the past. In the here discussed theory, for example, which is based on a strict time-reversal symmetry, the possibility of meaningfully asserting proposition Γ in the foregoing example depends on the fact that F is supposed to be valid. If, at t_2, we decide to measure not S_x but any other component of the spin, what then happens is not merely that we do not know any more whether or not proposition Γ was valid at t_1. Much more radically, it is that Γ becomes *meaningless*, or in other words that, at t_1, the particle ceases to *have* the definite S_x value it otherwise has, and this, just because of what we do at time t_2. In other words, the theory we have been discussing up to this point implies some influence of the future on the past. In a search for a "more realistic" description of things it is, as we just observed, natural to remove such influences from the theory. But, clearly, if we do we lose the possibility of interpreting the theory in question by saying in quite a general way that the physical quantities we measure already had the values we observe. In fact, we fall back on an interpretation very much akin to the "collapse" picture, with the kets interpreted as being the basic elements of reality. Needless to say that we are then faced with all the usual problems quantum measurement theory meets with.

2.4. *Third possibility*

It consists in accepting the view that two non-commuting operators can correspond to physical quantities having at the same time definite values on a system. But then the very concept of "consistent histories" loses a great part of its relevance. The resulting theory resembles somewhat the hidden variables theories ...

3. Conclusion

The consistent histories theory is thought-provoking and interesting. It is conceivable that it should lead to interesting theoretical or even mathematical developments. But those of us who look for an interpretation of quantum physics nearer than the orthodox view to our usual concepts about objects and the properties of objects should definitely not put too much of their hope on it.

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References

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[2] R.B. Griffiths, J. Stat. Phys. 36 (1984) 219.
[3] B. d'Espagnat, Found. Phys., to be published.